

# Low $\alpha_{strong}(M_Z)$ , Intermediate Scale SUSY SO(10) and Its Implications

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## Abstract

We show that one of the ways of obtaining consistency between the idea of supersymmetric grand unification and an apparent low value of  $\alpha_{strong}(M_Z) \simeq .11$  indicated by several low energy experiments is to have an intermediate scale corresponding to a local  $B - L$  symmetry breaking around the mass scale of  $10^{10}$  to  $10^{12}$  GeV. We discuss the realization of this idea within the framework of supersymmetric  $SO(10)$  grand unified theories with spectra of particles derivable from simple superstring-inspired versions of this model. We then study the  $b - \tau$  mass unification within this class of models and show that due to the influence of new gauge and Yukawa interactions beyond the intermediate scale, the prediction of the  $b$ -quark mass comes out well within the presently accepted values. We also discuss an un-orthodox class of SUSY models inspired by some theoretical considerations having two pairs of Higgs doublets at low energy and show that they also can lead to unification with intermediate scales and low  $\alpha_s(M_Z)$  as desired.

The high precision measurements of the gauge couplings  $\alpha_1, \alpha_2$  and  $\alpha_{strong}$  at LEP combined with the coupling constant evolution dictated by the minimal supersymmetric standard model, have in the past three years led to speculations that the present data while providing overwhelming support for the standard model may in fact be indicating that the next level of physics consists of a single scale supersymmetric grand unified theory, with new physics beyond supersymmetry appearing only at the scale of  $10^{16}$  GeV[1]. This has generated a great deal of excitement and activity in the area of supersymmetric grand unified theories(SUSY GUT). There are however reasons to believe that there could be new physics starting below  $10^{16}$  GeV, perhaps somewhere around  $10^{10}$  to  $10^{12}$  GeV or so based on attempts to understand the hot dark matter of the universe and to solve the strong CP problem etc. Since the idea of SUSY GUTs is so attractive, it is then important to ask whether an intermediate scale in the  $10^{12}$  GeV range fits within the framework of SUSY GUTs.

In this letter, we show that if the value of the QCD fine structure constant  $\alpha_s(M_Z)$  at the weak scale turns out to be around 0.11 as is indicated by several low energy experiments, then a simple and interesting way to reconcile it with the idea of SUSY GUTs is to have an intermediate scale around  $10^{10}$  to  $10^{12}$  GeV. Thus it may be argued that a lower value of  $\alpha_s(M_Z)$  may indeed provide an automatic reconciliation between the idea of SUSY GUTs and the idea of an intermediate scale motivated by reasons completely unrelated to it. There are of course scenarios where a low  $\alpha_s(M_Z)$  is consistent with intermediate scales other than the ones given above; we do not consider them here since they may be less well motivated from other considerations.

Before beginning the discussion of the intermediate scale GUT models, let us briefly review the situation with respect to the value of  $\alpha_s(M_Z)$ . A very useful summary of the issues have been given in a recent paper by Shifman[2], who has suggested that the discrepancy between the higher values of  $\approx 0.125$  for  $\alpha_s(M_Z)$  derived from LEP data on the one hand and the low energy data such as deep inelastic electron scattering, lattice calculations involving the upilon and the  $J/\Psi$  system etc on the other should be considered to be an indication of the presence of new physics[3]. If this new physics is identified with the supersymmetric version of the standard model(MSSM), then one can attempt to do a global fit to all LEP data and see if indeed the high value of  $\alpha_s(M_Z)$  indicated there is lowered. Such an analysis has been carried out recently by Kane, Stuart and Wells[4], who show that if the stop and the chargino masses in the MSSM are kept below a 100 GeV, then there are new contributions to the  $Z \rightarrow b\bar{b}$

decay which increase its decay width. In the presence of these contributions, the global fit to LEP data indeed leads to a value for  $\alpha_s(M_Z) \simeq .112$  which is what the low energy data give for this parameter. It should be noted that there are a number of other suggestions that could also lead to a higher value for the  $Z \rightarrow b\bar{b}$  width. It could very well be that one of these scenarios rather than the SUSY contribution is at the real heart of the problem. But for our discussion of unification, it is important that either the experimental value of the  $Z \rightarrow b\bar{b}$  come down or that the supersymmetric scenario provide an explanation for its enhancement over the standard model value.

The question then is what this implies for the unification of coupling constants. It is by now well-known that in the single scale unification theory, one predicts a value of  $\alpha_s(M_Z) \simeq .125$  or so. In a recent letter, Bagger et al[5] have shown that in the minimal SU(5) type theories, the inclusion of threshold corrections do not change this situation. Therefore one has to go beyond the simple single scale canonical GUT models to accommodate the lower value for  $\alpha_s(M_Z)$ .

One possibility is to espouse an alternative string type unification rather than the conventional SUSY GUT theory as has been done by Shifman and Roszkowski [6] and give up the unification of gaugino masses.

Our goal in this letter is to explore alternatives within the framework of the GUT hypothesis which can achieve the same goal i.e. accommodate a lower  $\alpha_s(M_Z)$  of about 0.11 while leading to eventual gauge coupling unification. We show that if a SUSY GUT theory based on SO(10) is allowed to have an intermediate scale in the interesting range of  $10^{10}$  to  $10^{12}$  GeV or so, one can construct scenarios which lead to  $\alpha_s(M_Z) \simeq 0.11$  keeping all other phenomenological implications consistent while adding the extra desirable features mentioned earlier.

In a general analysis of the SUSY SO(10) models with intermediate scales, it was noted recently that[7] that there are several instances where the intermediate scale is consistent with grand unification only if the value of  $\alpha_s(M_Z) \simeq .11$  or so. We pursue this alternative here and study the possibility of  $b - \tau$  unification in this class of models. In addition to confirming the results of [7], we obtain the following new results: (i) we show that  $b - \tau$  unification also prefers a lower value of  $\alpha_s$ ; (ii) we then present a new example of an unorthodox SUSY model at low energy which can lead to a grand unified SO(10) model with intermediate scale. The new model we have in mind is the one with two pairs of  $(H_u, H_d)$  in the low energy supersymmetric version of the standard model as against only one pair in the conventional MSSM.

These models have potentially richer phenomenology than the conventional MSSM and may have certain advantages in incorporating spontaneous CP violation into the supersymmetric standard model.

We will work within the class of  $SO(10)$  models suggested by the superstring theories with level two compactification[8], so that all particles assumed in our spectra arise from **10**, **16** +  $\overline{\mathbf{16}}$ , and **45** dimensional representations. The generic kind of theories we will be interested in will consist of two **10** dimensional, one or two **45** dimensional multiplets and several pairs of **16** +  $\overline{\mathbf{16}}$  at the GUT scale  $M_U$ . Only a small subset of these multiplets will be assumed to remain light below  $M_U$  down to the intermediate scale  $M_{B-L}$  and an even smaller subset below  $M_{B-L}$  down to  $M_W$ . We will define two classes of models depending on the particle spectra below  $M_{B-L}$ . Both the classes will of course contain the usual matter multiplets which will give the quarks and leptons of the three generations but will be distinguished by whether they have one or two pairs of Higgs doublet superfields:  $(H_u, H_d)$ . The model with one pair is of course the conventional MSSM and will be denoted as such; the model with two pairs will be called MSSM2. It is of course well known that MSSM2 is inconsistent with the idea of single scale grand unification but we will show that once we allow for the existence of an intermediate  $B - L$  symmetry scale, it is in fact consistent with a lower  $\alpha_s(M_Z)$  which is the main focus of this paper. Let us consider these cases separately.

### I. The MSSM case:

It was pointed out in Ref.[7] that under the above conditions, the only particle spectrum between  $M_U$  and  $M_{B-L}$  which can lead to  $\alpha_s(M_Z) \simeq 0.11$  is the one with two pairs of  $(2, 2, 0, 1)$  and five pairs of  $(1, 2, 1, 1) + (1, 2, -1, 1)$  multiplets under the group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ . Our calculations have confirmed this result and we find in the one loop approximation that  $\alpha_s(M_Z) = 0.11$ ,  $M_U \simeq 2.05 \times 10^{15}$  GeV and  $M_{B-L} \simeq 3.47 \times 10^{12}$  GeV. As shown in Ref.[7], inclusion of the two loop corrections change this result only very slightly. The graph displaying the evolution of the three gauge couplings in the one loop approximation is given in Fig.1. The lower value of the  $M_U$  leads to a gauge mediated proton lifetime ( for the decay mode  $p \rightarrow e^+ \pi^0$  ) of  $.62 \times 10^{33 \pm .7}$  years up to threshold corrections. It is interesting that this is within the reach of the Super Kamiokande experiment. The Higgsino mediated proton decay amplitude in this case must be suppressed by some additional mechanism[9]

Let us now address the question of predicting the mass of the top and the bottom quark masses in these theories: the first by using the idea of infra-red fixed point of the top Yukawa

coupling assumed to be large at the GUT scale [10] and the second by using the property of these models that at the GUT scale, one has  $m_b(M_U) = m_\tau(M_U)$ . In this class of models where the  $B - L$  symmetry is broken at the intermediate scale, the conventional approach to both these questions will be modified [11, 12, 7] due to new evolution equations for the Yukawa couplings above  $M_{B-L}$ .

It is widely known that in SUSY GUTs with one step breaking predict a large value of  $m_b$  for the major part of the parameter space [13]. The study of  $b$ - $\tau$  unification including a right handed neutrino has also been performed [14]. However, in this study no new gauge interactions beyond the intermediate scale has been considered and due to renormalization effects of the new Yukawa coupling, a 10-15 % increase in the mass of the  $b$ -quark was obtained. We will show that after the inclusion of the new left-right symmetric gauge and Higgs interactions at  $M_{B-L}$  surviving from an  $SO(10)$  GUT, the running of the  $b$ -quark Yukawa coupling is altered and as a result an attractive reconciliation with the experimental measurements can be achieved. To discuss these, let us recall that since there are two Higgs bidoublets (denoted by  $\phi_1$  and  $\phi_2$ ) above  $M_{B-L}$ , the Yukawa superpotential of the quarks and leptons are given by:

$$\mathcal{W}_Y = h_{Q_1} Q^T \tau_2 \phi_1 Q^c + h_{Q_2} Q^T \tau_2 \phi_2 Q^c + h_{L_1} L^T \tau_2 \phi_1 L^c + h_{L_2} L^T \tau_2 \phi_2 L^c + h.c. \quad (1)$$

(where we have denoted the quarks and leptons by the obvious notation  $Q, Q^c$  and  $L, L^c$ ). At the GUT scale, we have  $h_{Q_1}(M_U) = h_{L_1}(M_U)$  and  $h_{Q_2}(M_U) = h_{L_2}(M_U)$ . It is possible to construct doublet triplet splitting mechanisms [15] where the  $H_u$  and  $H_d$  arise from  $\phi_1$  and  $\phi_2$  separately. In this case, the model has  $b$ - $\tau$  unification at the GUT scale and we can study whether the bottom mass is predicted correctly at low energies. The evolution equations above  $M_{B-L}$  in this case are given by [7]. We are rewriting them in the notation of [16], as,

$$\frac{dY_a}{dt} = Y_a [S_{ab} Y_b - Z_{ai} \alpha_i]. \quad (2)$$

The variable  $t$  is given by,

$$t = \frac{1}{2\pi} \ln \frac{\mu}{GeV}, \quad (3)$$

where  $\mu$  denotes the renormalization scale, and,  $S$  is a  $4 \times 4$  matrix and  $Z$  is a  $4 \times 4$  matrix given by:

$$S = \begin{pmatrix} 7 & 4 & 1 & 0 \\ 4 & 7 & 0 & 1 \\ 3 & 0 & 5 & 4 \\ 0 & 3 & 4 & 5 \end{pmatrix}, \quad (4)$$

and

$$Z = \begin{pmatrix} 1/6 & 3 & 3 & 16/3 \\ 1/6 & 3 & 3 & 16/3 \\ 3/2 & 3 & 3 & 0 \\ 3/2 & 3 & 3 & 0 \end{pmatrix}. \quad (5)$$

In the above equations, we have chosen the basis  $(Y_{Q_1}, Y_{Q_2}, Y_{L_1}, Y_{L_2})$  with  $Y_i = h_i^2/4\pi$  and the basis for the fine structure constants is the one corresponding to  $(U(1)_{B-L}, SU(2)_L, SU(2)_R, SU(3)_c)$ . We point out that below the scale  $M_{B-L}$  the right-handed neutrino contributions must be absent since this particle decouples from low energy physics. The equations for the Yukawa couplings also change and acquire the standard form [10]. We have numerically solved these equations for a large range of values for  $Y_{Q_2} = Y_{L_2} = Y_2$ . We have kept the top Yukawa coupling  $Y_{Q_1} = Y_{L_1} = Y_1$  at  $M_U$  to be equal to 1 and obtained in different cases the values for the ratio  $R_{b/\tau}(M_t) \equiv m_b(M_t)/m_\tau(M_t)$  which are summarized in Table. 1. We point out that for  $\alpha_s(M_Z) \simeq 0.11$ , the low energy values of  $m_b$  and  $m_\tau$  when extrapolated to the scale  $M_Z$  gives an  $R_{b/\tau}(M_t) \simeq 1.7$  to 1.9 for values of  $m_b(m_b) = 4.1$  to 4.5 GeV. Thus the values of  $m_b$  obtained by us in these models are quite consistent. The corresponding predictions for the top mass  $m_t(pole) = m_t(M_t) [1 + \frac{4\alpha_s}{3\pi}]$  lies between 177 to 199 GeV which again is also consistent with data. The value of  $\tan\beta$  is determined from the measured value of the tau lepton mass. In Fig.2, we display the evolution of the Yukawa couplings for the case of  $Y_{Q_2}(M_U) = Y_{L_2}(M_U) = 0.1$  as typical example.

$Y_1$	$Y_2$	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$\tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
1	1	1.010	0.96	0.62	60.43	2.77	176.83	1.56
1	$10^{-1}$	1.060	0.84	0.52	51.26	2.85	184.46	1.60
1	$10^{-2}$	1.094	0.460	0.270	26.7	3.01	190.34	1.69
1	$10^{-3}$	1.103	0.160	0.095	9.25	3.06	190.90	1.72
1	$10^{-4}$	1.104	0.054	0.030	2.80	3.07	181.03	1.73
1	$\frac{1}{2}10^{-4}$	1.104	0.037	0.021	1.85	3.08	169.16	1.73

Table 1: The values of  $h_t(M_t)$ ,  $h_b(M_t)$ ,  $h_\tau(M_t)$  and calculated by RGE for  $\alpha_s = 0.11$  in the MSSM case. The prediction of the masses  $m_b$  and  $m_t$  at the scale  $M_t$  has been quoted in GeV.  $M_t$  is defined as 170 GeV.  $\tan\beta$  has been calculated assuming  $m_\tau(M_Z) = 1.777$  GeV.

## II. The MSSM2 Model:

In this section, we present the new possibility for the low energy supersymmetric model which also leads to a lower  $\alpha_s(M_Z)$ . For this model, the particle spectra below  $M_{B-L}$  consists of an extra pair of Higgs doublet superfields ( $H_u, H_d$ ) in addition to the particles already present in the MSSM. We call this model MSSM2 from now on. Above the scale  $M_{B-L}$ , we assume the additional particles to be: one (1,1,0,8) multiplet, four pairs of (1,2,1,1) + (1,2,-1,1), two bidoublets (2,2,0,1) in the basis  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ . Below the  $M_{B-L}$  scale, we assume this theory to consist of two pairs of the Higgs doublets of MSSM2. Note that the (1,1,0,8) multiplet could arise from the SO(10) multiplet **45**. We then find that there is coupling constant unification with  $\alpha_s(M_Z) = .11$ ,  $M_{B-L} \simeq 1.52 \times 10^{11}$  GeV and  $M_U \simeq 1.72 \times 10^{16}$  GeV. All these numbers are evaluated in the one loop approximation; we expect them to change by 1 or 2% due to two loop corrections. The one loop running for this case is shown in Fig.3. The gauge mediated proton decay in this case is suppressed as in the usual single scale GUT models.

Before exploring the implications of  $b - \tau$  Yukawa unification in this case, we assume that due to some discrete symmetry, below  $M_{B-L}$ , only one pair of the left-handed doublets remains coupled to the fermions and another pair just "floating around" to help with soft CP violations etc. The rest of the discussion similar to the one in the previous section except that here we have a different evolution of the gauge couplings above the  $M_{B-L}$  scale. We have also numerically analyzed the Yukawa evolution equations in this case; we give it in Fig.4 and again, we find that the bottom quark mass comes out in very good agreement with experiment and the predicted value of  $m_t(M_t)$  is given in Table 2.  $m_t(pole)$  is in the range 173 to 198 GeV in this case.

These models may also have advantages for introduction of spontaneous CP violation[17] and have a potentially rich Higgs phenomenology[18]. We hope to explore these questions.

We now briefly discuss a model, which we call MSSM3, that reduces to MSSM2 below  $M_{B-L}$ . The model we present arises by adding an extra pair of (2,1,1,1)+(2,1,-1,1) multiplets, above the scale  $M_{B-L}$ . Denoting them by  $\chi_L + \bar{\chi}_L$ , and the corresponding righthanded doublets by  $\chi_R + \bar{\chi}_R$ , we can have couplings of the form  $\bar{\chi}_L \bar{\chi}_R \phi$  ( where  $\phi \equiv (2, 2, 0, 1)$ ). Let us choose the relevant  $\phi$  to be the  $\phi_1$  of Eq.1. Once  $\bar{\chi}_R$  acquires a vev the down type doublet in the  $\phi$  pairs up with  $\bar{\chi}_L$  to acquire mass of order  $M_{B-L}$  and the accompanying  $\chi_L$  which is uncoupled to quarks and leptons remains as an  $H'_d$  at low energies. Now note that in Eq.1, the  $\phi_2$

$Y_1$	$Y_2$	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$\tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
1	1	0.99	0.960	0.59	57.81	2.87	173.90	1.61
1	$10^{-1}$	1.05	0.830	0.50	48.78	2.97	182.65	1.67
1	$10^{-2}$	1.09	0.460	0.26	25.41	3.17	189.11	1.78
1	$10^{-3}$	1.10	0.160	0.09	8.81	3.23	189.69	1.82
1	$10^{-4}$	1.10	0.052	0.023	2.65	3.24	178.80	1.82
1	$\frac{1}{2}10^{-4}$	1.104	0.037	0.020	1.74	3.24	165.70	1.82

Table 2: The values of  $h_t(M_t)$ ,  $h_b(M_t)$ ,  $h_\tau(M_t)$  and calculated by RGE for  $\alpha_s = 0.11$  in MSSM2 case. The prediction of the masses  $m_b$  and  $m_t$  at the scale  $M_t$  has been quoted in GeV.  $M_t$  is defined as 170 GeV.  $\tan\beta$  has been calculated assuming  $m_\tau(M_Z) = 1.777$  GeV.

couplings lead to bottom quark mass and we can choose this coupling to be small ( for small  $\tan\beta$ ). Even though the  $\phi_2$  contains the second  $H_u$  of MSSM2, because of its small Yukawa coupling, it leaves the Yukawa evolution discussion of the previous section remains unchanged (except for the effect of changes in the evolution of the gauge couplings). We have studied the Yukawa and gauge coupling evolution in this model and they are given in fig. 5 and 6 and some representative numbers in the table 3. In this model, the value of  $M_U \simeq 3.55 \times 10^{15}$  GeV and the value of  $M_R \simeq 4.81 \times 10^9$  GeV. The Yukawa unification also works in this case. Further details of the model will be the subject of a longer paper under preparation.

$Y_1$	$Y_2$	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$\tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
1	$10^{-4}$	1.104	0.050	0.027	2.51	3.23	178.23	1.81
1	$\frac{1}{2}10^{-4}$	1.104	0.035	0.019	1.63	3.21	163.57	1.81

Table 3: The values of  $h_t(M_t)$ ,  $h_b(M_t)$ ,  $h_\tau(M_t)$  and calculated by RGE for  $\alpha_s = 0.11$  in the MSSM3 case. The prediction of the masses  $m_b$  and  $m_t$  at the scale  $M_t$  has been quoted in GeV.  $M_t$  is defined as 170 GeV.  $\tan\beta$  has been calculated assuming  $m_\tau(M_Z) = 1.777$  GeV.

In conclusion, we have shown that a simple way to reconcile a lower value of  $\alpha_s(M_Z)$  with the SUSY GUT hypothesis is to admit the existence of an intermediate scale corresponding to local  $B - L$  symmetry. While the value of the intermediate scale depends on the choice of



light Higgs multiplets, we have focussed on scenarios where this scale is around  $10^{10}$  to  $10^{12}$  GeV or so since they have other independent physical motivations. We have outlined three classes of scenarios which achieve this goal; one with the usual MSSM structure at low energies and a second one which has two pairs of Higgs doublets in the low energy domain. We have also shown that such gauge and Higgs structures above the intermediate scale predicted by the unification of gauge couplings influences the running of the b quark Yukawa coupling in the proper direction. As a result, a correct prediction of the  $m_b(m_b)$  can be obtained which is otherwise difficult to achieve in MSSM with single scale GUT. Furthermore, our scenario also predicts top quark mass in the appropriate range when the top quark Yukawa coupling is in its fixed point domain at the unification scale. We have also outlined a scenario where one can realize the scenario of MSSM2 in a gauge model.

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## Figure Captions

Figure 1. The evolutions of various  $\alpha_i^{-1}$  and the unification of gauge couplings in model 1. We have chosen  $\frac{1}{2\pi} \ln \frac{\mu}{GeV}$  as the variable represented along the x-axis in this and all subsequent figures.

Figure 2. The evolution of Yukawa couplings in model 1. The different lines from top to bottom represent the evolution of  $Y_t, Y_\nu, Y_b$  and  $Y_\tau$  respectively.

Figure 3. The evolution of various  $\alpha_i^{-1}$  and the unification of gauge couplings in model 2.

Figure 4. The evolution of Yukawa couplings in model 2 with the lines representing the cases as in Fig.2 .

Figure 5. The evolution of the various  $\alpha_i^{-1}$  and the unification of gauge couplings for MSSM3.

Figure 6. The evolution of Yukawa couplings in a typical case for MSSM3.

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